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Some Aspects of Beam Waveguides for Long Distance Transmission at Optical Frequencies

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Summary—Two types of beam waveguides are discussed in this paper, the iris-type and the lens-type. Both appear applicable to guided long distance transmission of light with theoretical losses of less than 1 db/km. However, there are problems concerning their practicability which require experimental investigation. Such problems are the alignment of the irises or the lenses, the effects of turbulence and stratification of air along the light path, and the required tolerance in the construction of the lenses. Since the lens-type guide offers a simple possibility for compensating misalignments, an experimental waveguide of this type has been constructed, having a length of approximately 1 km and comprising 10 iterations. The light path is enclosed by a 4 inch aluminum pipe which is supported within a 6 inch aluminum pipe. The first series of experiments which is reported in this paper indicated that there are no serious alignment problems. However, it was found that the effects of turbulence and air stratification are usually very severe and it appears necessary to provide an evacuated light path to obtain constant transmission conditions. It was also found that the available lenses add considerably higher iteration loss than expected. This increased loss was primarily caused by inadequate surface coating. A theoretical study of beam propagation in a misaligned lens-type guide is included in the Appendix.

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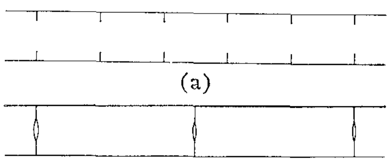
Introduction

With the recent development of the optical maser (laser), an extremely large frequency range has been made available to communications. Unfortunately, the utilization of this frequency range is seriously hampered by the vulnerability of light propagation through the atmosphere not only by fog, rain or snow, but also by turbulence. An obvious way to overcome this difficulty is to provide a protected light path. Efficient long-distance transmission would, however, not result if the light beam were simply passed through an ordinary pipe. Although it is possible to produce coherent optical beams of extremely small divergence, it would be quite expensive to provide pipelines which were so straight that the beam would not hit the wall within a distance of a few hundred meters. Reflections on the wall of the pipe would not only cause substantial transmission loss, but also severe delay distortions. Eaglesfield [1] discusses the possibility of transmitting light through a pipe of precision bore whose inner surface has a mirror finish. In this case, light is propagated by multiple internal reflections. Although the theoretical loss is quite small, there are inherent multipath distortions caused by the many simultaneous transmission paths.

This paper discusses some aspects of the application of beam waveguides to long-distance transmission of light. The beam waveguide which has been developed originally for millimeter and submillimeter waves [2]–[4] differs from conventional waveguides in that the propagated field is not a wave mode in the usual sense, but a wave beam whose field configuration is similar to that in confocal laser resonators.

General Properties of Beam Waveguides

The two kinds of beam waveguides considered in this paper are illustrated schematically in Fig. 1. The "iris-type" simply consists of uniformly spaced irises, while the "lens-type" comprises in addition lens-shaped dielectric phase correction plates. Both are based on the existence of so-called "reiterative wavebeams"; *i.e.* wavebeams whose field distribution is re-established or iterated at periodic intervals. In the iris-type guide, the iteration is accomplished by *diffraction*. The beam, when passing through an iris, is somewhat restricted in diameter by the aperture of the iris. The associated field distortion compensates for the field transformation that takes place during the journey of the beam from one iris to the next. In the lens-type guide, the iteration is primarily by *refraction*. Each lens resets the cross-sectional phase distribution in the beam to the distribution which exists at the preceding lens. Since the lenses do not perform as geometric optical image-forming devices, the names "phase correction plates" or "phase transformers" are more appropriate from the functional point of view. Diffraction by the lens apertures has little effect on the iteration process though it contributes to the iteration loss.



(a)

(b)

Fig. 1—Schematic of, (a) Iris type, (b) Lens type, beam waveguide.

In both cases very low iteration loss can be achieved if the apertures are sufficiently large compared to the wavelength. This is apparent from the curves in Figs. 2–4. Fig. 2 shows the theoretical diffraction loss per iteration of the fundamental beam modes in the two types of beam waveguides as a function of the aperture area divided by the product of wavelength and iteration path length D. The curves have been verified experimentally at various frequencies [4]–[6] although not within the optical range; but there is no reason why they should not be applicable to this range also. The curves in Figs. 3 and 4 are derived from those in Fig. 2, and show the relationship between the diffraction loss of a beam waveguide in db/km vs iris or lens spacing D respectively, for constant aperture radii R measured in terms of wavelengths. For each parameter n, the curves cover a R/λ range of 1 to 10. In the iris case (Fig. 3), it is uncertain whether the curves hold for R/λ smaller than, say, 25, since measurements have been made only for irises with radii greater than 30λ. In regard to the lens-type waveguide, it should be mentioned that the data pertain to the condition of minimum field extension at the lens aperture, where the focal length of the lenses is one half their spacing D. For any other focal length, the loss per unit length is increased by the factor

$$F = \frac{f/D}{\sqrt{f/D - 1/4}}$$

because the spacing of the lenses must be smaller by a factor $1/F$ if the same field extension at the lenses is assumed [7], [8].

The iteration loss in the lens-type guide is, of course, increased by the inherent reflection and absorption losses of the lenses. In fact, these losses determine the minimum obtainable transmission loss since the diffraction loss can be made almost arbitrarily small. This is apparent from the steepness of the curves $R/\lambda = \text{constant}$ in Fig. 4. If the radius is increased by only a few per cent, a large decrease in loss per km results, thus rendering the diffraction loss negligible. With modern techniques reflection and absorption losses can be limited to less than 1/10 db per lens.

The iris-type waveguide is not practical in the millimeter wave range, because it requires an extremely large aperture to wavelength ratio in order to be efficient. This fact is illustrated by the following example. Assuming a wavelength of $\lambda = 1$ mm, an iris spacing of $D = 10$ m or $10^4\lambda$ and a permissible transmission loss of $L = 1$ db/km, one obtains with the appropriate curve of Fig. 3, using the set parameter $n = 2$, an iris radius of $R = 500\lambda$ or 50 cm. For the lens-type waveguide with the same D and L, the curves of Fig. 4 with $n = 1$ yield a lens radius of only 95λ or 9.5 cm. If the radius is increased by 5 per cent the diffraction loss in the iris-type guide is reduced to 0.8 db/km and in the lens-type guide to 0.1 db/km which is negligible compared to the dielectric losses of the lenses.

In the optical frequency range where very large R/λ values are easily realized, the iris-type guide merits consideration. For example, assuming $\lambda = 1$ micron, $D = 10$ m or $10^7\lambda$ and a permissible loss of 1 db/km, one obtains from Fig. 3 ($n = 4$) an iris radius of $R = 1.7 \times 10^4\lambda = 1.7$ cm, which is quite reasonable in size. A lens-type guide with the same spacing requires, according to Fig. 4 ($n = 3$), only an aperture radius R of $3 \times 10^3\lambda = 3$ mm. This diameter is quite small, and it appears reasonable to increase the spacing. For $D = 100$ m and the same diffraction loss of 1 db/km, the value of R becomes 8.2 mm. If this radius is increased to 10 mm or more, the diffraction loss is less than 10^{-2} db/km, in which case the diffraction loss is negligible.

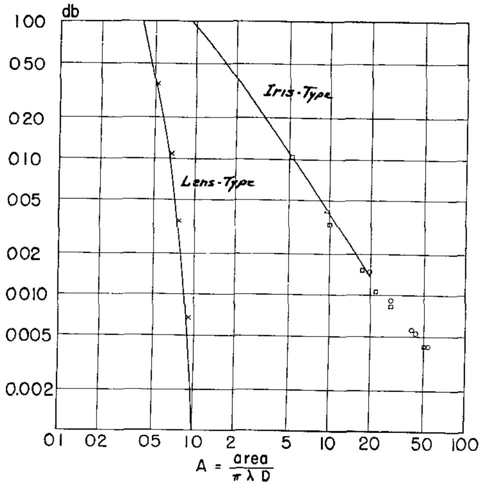


Fig. 2—Theoretical and measured diffraction loss per iteration of the fundamental beam modes of the iris-type and the lens-type beam waveguides.
measured values—
×—by Scheibe [11] and Beyer and Scheibe [12]
□○—by Christian and Goubau [6]
△—by King, *et al.* [13]
theoretical curves—
lens-type—Goubau and Christian [2]
iris-type—Fox and Li [14].

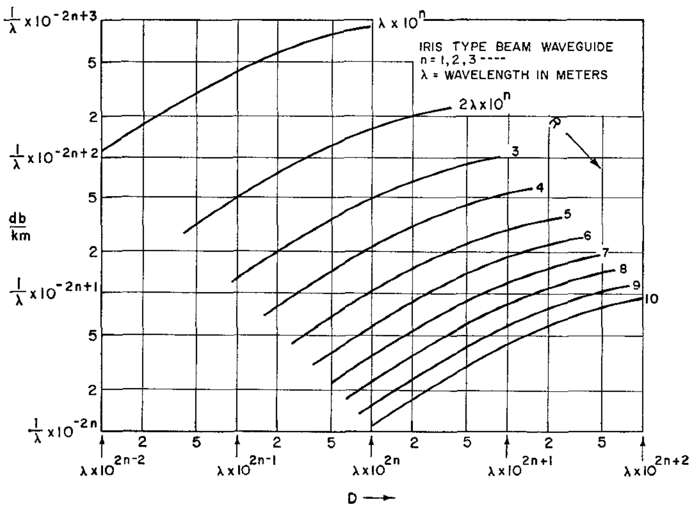


Fig. 3—Diffraction loss of an iris-type beam waveguide in db/km vs iris spacing, for constant aperture radii.

An important problem regarding the operation of beam waveguides is the alignment of the apertures. There is very little known about the beam propagation in a misaligned iris-type guide. Since iteration is solely caused by diffraction at the apertures, misalignment should impair the operation more seriously than in the lens-type guide where iteration is performed by refraction. The misalignment of a lens causes essentially only a deflection of the beams by the amount

$$\alpha = s/f$$

where s is the displacement of the lens and f the focal length (see Fig. 5). The beam then proceeds along a zig-zag path whereby the lateral displacement remains

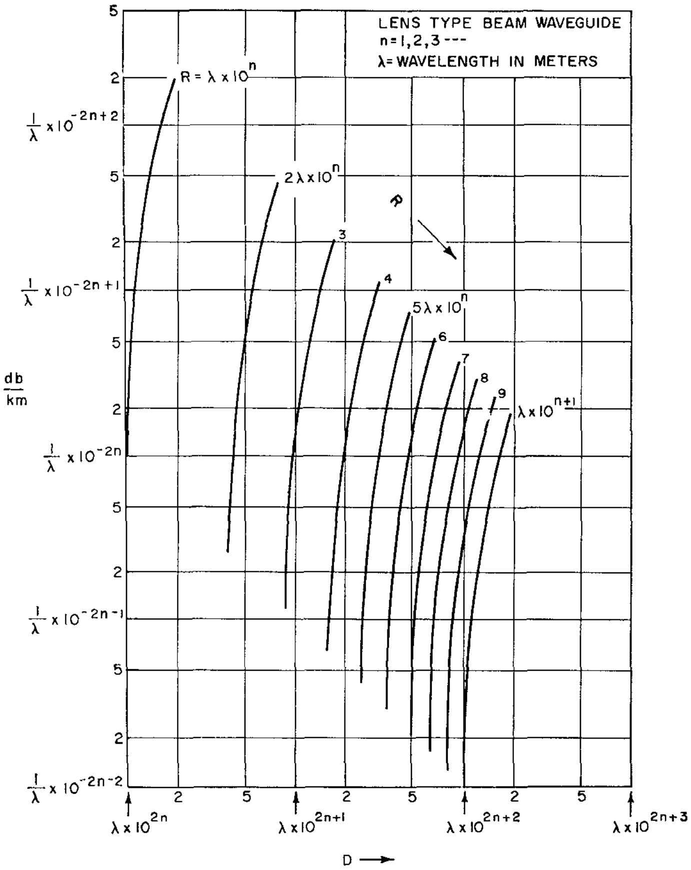


Fig. 4—Diffraction loss of a lens-type beam waveguide in db/km vs lens spacing, for constant aperture radii.

within determinable limits (see the Appendix). Fig. 6 shows the zig-zag path in the case $D = 2f$. If more lenses are randomly displaced, the limits of maximum excursion of the path increases randomly with the number of displaced lenses. As long as the diameters of the lenses are large enough to accommodate the excursions of the beam, no increase in loss is effected. The beam deflection by lateral displacements of the lenses can even be applied to compensate for small curves in the layout of the guide. Large bends, of course, must be accomplished by prisms or reflectors. It should be mentioned that small tilts of the lenses or irregularities in their spacing have no appreciable effect on the transmission loss

The greater sensitivity of the iris-type guide to misalignment has been experienced in measurements in the millimeter wave range [6], and has also been observed in laser resonators [9], [10]. The mode systems of the parallel mirror and the confocal mirror resonators are essentially the same as those of the iris-type guide and the lens-type guide, respectively.

Although there is no doubt from the theoretical point of view that beam waveguides are applicable to the transmission of coherent light, there are several practical questions which can only be answered experimentally. In both cases, it is only necessary to provide stable supports for the irises or the lenses while the intercon-

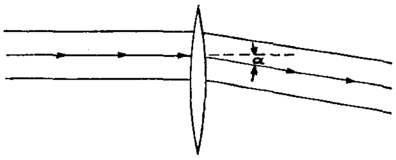


Fig. 5—Deflection of beam caused by lens displaced from beam center.

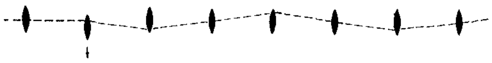


Fig. 6—Zig-zag path of beam when one lens is laterally displaced, for the case $D=2f$.

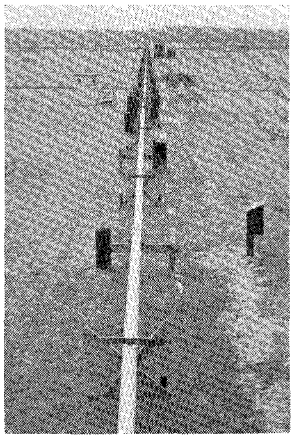


Fig. 7—Photograph of the experimental beam waveguide.

necting pipe may have bends or sags as long as the diameter is large enough not to obstruct the beam. At first sight, the iris-type guide appears simpler than the lens-type guide. However, for competitive beam diameters, the number of stable supports would have to be much greater for the iris-type guide, since many more apertures are required to achieve the same transmission loss. For competitive iteration lengths, the much larger beam diameter of the iris-type guide would create difficulties in launching the low-loss mode, and also render the guide more susceptible to turbulence of the air along the light path. Considering, furthermore, the more stringent alignment requirements for the iris-type guide, it was concluded that the lens-type guide should be less expensive in construction and offer a better chance for success. It was, therefore, decided to construct a simple experimental model long enough to render representative information about long distance transmission.

The questions of primary interest are the following: 1) Can the alignment of the phase transformers be practically realized and maintained, considering the fact that any support, no matter how rigidly constructed, will be subjected to movements of the ground? 2) If the guide is filled with air or any other gas, how serious is the effect of turbulence in the gas on the propagation, and what are the tolerable temperature and pressure gradients to keep the turbulence at an acceptable level? 3) What precision is required in the manufacturing of the phase transformer units; in particular, what tolerance is acceptable in the manufacturing of the lenses? 4) What minimum transmission loss can be practically achieved?

Experimental Model

The experimental model described in the following was not designed to answer all the above questions, but only to obtain orientative information as to the requirements for a workable model of a long-distance transmission system. A photograph of the experimental layout is shown in Fig. 7. The entire light path of 970 m length is enclosed by a 4-inch aluminum pipe which is supported inside a 6-inch aluminum pipe to reduce temperature variations and turbulence along the light path. The outer pipe is supported by utility poles. The height above ground varies between 2 to 7 feet in order to provide a straight light path which is easier to align than one with bends. Iteration is performed at intervals of 97 m using lenses of approximately 48 m focal length. A phase transforming unit with its cover removed is shown in Figs. 8 and 9. The lens is mounted in a frame which can be moved laterally in two perpendicular directions by means of rotary stepping solenoids with suitable gearing mechanism in order to produce movements of the lens in steps of 0.05 mm. The movements are remotely controlled to simplify the alignment procedure. The units also contain a viewing device comprising a movable mirror to deflect the beam to the side for observation.

The ideal source for a beam waveguide is a laser operating with a confocal resonator whose fundamental mode is essentially the same as the mode propagated in the beam waveguide. The mode parameters in both cases differ, however, since the focal lengths of the laser reflectors and the lenses of the beam waveguide are different. The transformation of the mode parameter is easily accomplished by means of appropriate lenses [7].

Since CW lasers were not yet in existence when the experiments were planned and, furthermore, since the primary purpose of the experiment was to obtain orientative information, an incoherent light source was used. It is well known from elementary optical diffraction experiments that a small amount of coherent light can be derived from an incoherent source. It can be shown that the amount of power which is available for a single beam mode is $\lambda^2/2\pi$ times the surface density of the radiated power. Although this amount is quite small, it proved to be adequate.

A high-pressure mercury lamp was first used because of the greater brightness; however, the inherent movements of the arc were too large to obtain a stable beam. Therefore, the mercury lamp was replaced by an ordinary 6 w tungsten lamp, combined with a collimator lens of 1 m focal length and an optical filter to pass only part of the spectrum. The source, mounted at a dis-

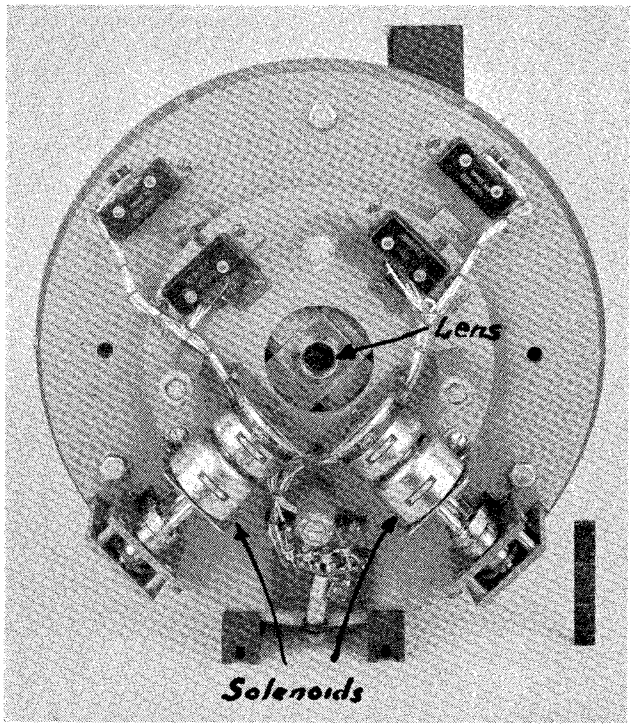


Fig. 8—Phase transforming unit with cover removed showing lens and moving mechanism.

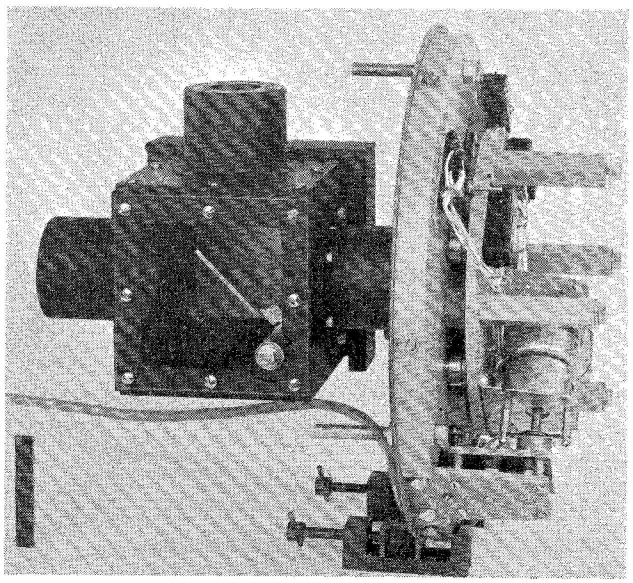


Fig. 9—Side view of phase transforming unit showing viewing device used for beam observation.

tance of approximately 48 m (half the iteration length) from the first phase transformer, was modulated with 1000 cps by means of a mechanical chopper. The aperture of the first lens of the guide was reduced to $\frac{1}{4}$ inch to insure an essentially homogeneous phase front within the aperture of this iris, and thus, a space coherent beam. Of course, the beam did not have the field distribution of the low-loss fundamental beam mode. In other words, it comprised a relatively large number of beam modes. However, since the higher modes have higher iteration losses, the lower mode prevailed at the end of the guide.

The receiver consisted of an RCA photomultiplier tube type 7265 in conjunction with a band-pass amplifier tuned to the modulation frequency of the source.

Results and Conclusions

In spite of the double shielded light path, the effect of air drifts and temperature stratification on the beam propagation were severe. On clear days when the pipe line was exposed to direct sun radiation, the continuously varying deflections of the beam were so great that it was almost impossible to maintain any kind of alignment. On cloud covered days, the deflection of the beam could be compensated by appropriate positioning of the lenses. However, the variations of the output signal level at the end of the guide were still very large. A constant signal was obtained only during certain periods of the night when temperature variation was small. Fig. 10 shows a sample recording during steady conditions taken at the 8th, 9th and 10th lenses, subsequently. The fluctuations in this case were below 0.1 db and essentially caused by noise and intrinsic variations in the equipment.

The alignment of the lenses during quiescent conditions presented no difficulty. After approximate alignment was achieved, the lenses were adjusted remotely for maximum signal output. This optimization procedure is not the most ideal one in that it can lead to a curved transmission path of the kind discussed in Case 3 of the Appendix. Although such a path has practically the same transmission loss as the straight path, precaution must be taken in order that after several realignments which might be necessary over longer periods of time (days or weeks) the curvature is not gradually increased to the extent where the beam is restricted by the apertures of the lenses. This difficulty can be eliminated if another and more straightforward alignment procedure is used as discussed in Case 4 of the Appendix. This procedure requires a probing device at each transformer unit which establishes the beam position relative to the lens. Such a device was, however, not available for the experiments discussed in this paper. The measurement of the transmission loss offered a number of problems. Since an incoherent source was used, the low-loss mode was not completely developed even at the end of the guide. Beam intensity measurements were, therefore, performed only at the 8th, 9th and 10th lenses. These measurements had to be made without disconnecting the photomultiplier from its power supply since the period of constant propagation conditions were usually not more than one half hour while the "warm-up" time of the photomultiplier assembly exceeded 10 minutes.

The recording in Fig. 10 indicates a drop in signal level of 0.4 db from lens 8 to lens 9 and 0.9 db from lens 9 to lens 10. Exchanging lenses showed that the iteration loss of the lenses varied between 0.4 and 0.9 db.

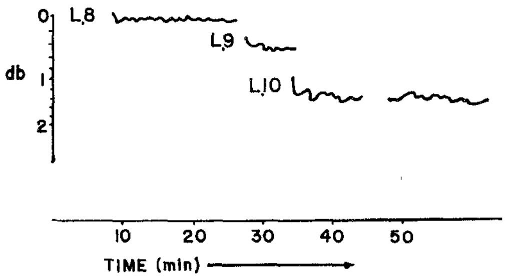


Fig. 10—A reproduction of a sample recording of signal intensities measured behind the 8th, 9th and 10th phase transforming units.

These losses are substantially higher than expected. Although part of the increased loss may be due to the presence of higher modes, it is essentially caused by imperfection of the lenses. Measurements of the inherent transmission loss of each lens (reflection absorption and scatter) yielded values between 0.3 and 0.35 db, depending on the spectral range. Since the inherent losses should not be greater than 0.1 db, the reflection coating of the lenses was apparently very poor. This was also verified by visual inspection. The added iteration loss, in some lenses up to 0.6 db, must have been caused by poor phase transformation which, in turn, could have been caused by inhomogeneities in the glass, in the coating, or by irregularities of the ground surfaces. Of course, there is a possibility that some of the added iteration loss is attributable to distortions of the phase fronts by stratifications of the air. This possibility is borne out by the observation that measured loss varied somewhat from day to day.

In order to eliminate atmosphere effects completely and to establish true iteration losses, the present set-up is being modified to provide the possibility of evacuating the entire beam path. New lenses will be installed with special attention being given to their quality, particularly in regard to their homogeneity, and their coating. A CW gas laser will be used instead of an incoherent source so that the low-loss mode can be launched immediately and the entire length of the guide is made available for quantitative measurements. The laser will also provide the possibility of performing measurements in the infrared region as well as in the visible region. In this manner information as to the precision of the lens grinding can be obtained since any phase error caused by nonspherical surfaces is inversely proportional to the wavelength and should, therefore, be less severe in the infrared.

Appendix

In this Appendix the following alignment problems shall be discussed: 1) The laser beam and all but one lens of the guide are aligned. What is the path of the beam after it passes the misaligned lens? 2) The guide is aligned, but the laser is out of alignment. 3) Assume the laser is not axial with the ideal transmission path, how should the lenses be aligned for optimum transmission, assuming the last lens is fixed in location? 4) Assume there is a probing device to establish the position of the beam axis relative to the optical center of the lens, how can the optimum alignment be accomplished?

Although the assumption is being made that the desired path of the beam is a straight path, the following considerations apply also to a beam waveguide which comprises deflecting prisms or mirrors, so that the desired path is not a straight one. The misalignment and the deviations of the beam would then refer to the ideal path for this particular set-up. We consider only misalignments in one plane; it is obvious that misalignment in two perpendicular planes can be treated independently.

First, we derive the general relations for the path of a beam in a misaligned guide, assuming that misalignments are sufficiently small so that the location of the center of the beam (maximum power density) is not substantially affected by diffraction effects caused by the apertures of the lenses.

Fig. 11 illustrates schematically a poorly aligned beam waveguide. The incident laser beam is assumed to arrive at lens No. 1 with a field distribution corresponding to the beam mode which is propagated along the guide.

In order to establish a reference from which lens displacements and beam excursions are measured, we connect an arbitrary point within the aperture of the first lens, and the center of the last lens N with a straight line.

The excursions of the beam at the locations of the lenses relative to the reference line are denoted by

$$y_1, y_2, y_3, \cdots, y_N;$$

the angles of the beam axis vs the reference line at the incident sides of each lens by

$$\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_N;$$

and the distances between beam centers and lens centers by

$$s_1, s_2, s_3, \cdots, s_N.$$

The displacements x_n of the centers of the lenses from the reference line are then

$$x_n = y_n - s_n. \tag{1}$$

The angle deflection δ_n of a beam which passes a lens at a distance S_n from the lens center is

$$\delta_n = -\frac{s_n}{f}, \tag{2}$$

where f is the focal length of the lens. The minus sign indicates that the lens deflects the beam toward the lens axis.

The beam arriving at lens n with the angle α_n proceeds to the following lens $(n+1)$ with the angle

$$\alpha_{n+1} = \alpha_n + \delta_n = \alpha_n - \frac{s_n}{f}. \tag{3}$$

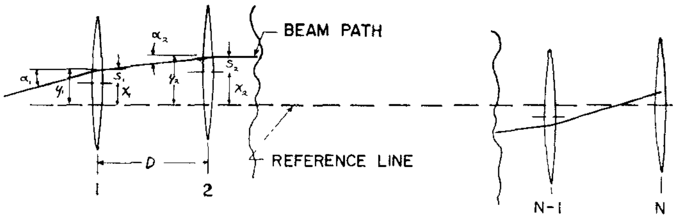


Fig. 11—Beam path in a poorly aligned guide.

The beam coordinate at this lens is

$$y_{n+1} = y_n + \alpha_{n+1} \cdot D, \tag{4}$$

where D is the spacing between the lenses.

If we express y_n by the beam coordinate y_{n-1} at the preceding lens, we obtain a similar equation

$$y_n = y_{n-1} + \alpha_n \cdot D. \tag{5}$$

Subtracting (5) from (4) yields

$$y_{n+1} - y_n = y_n - y_{n-1} + (\alpha_{n+1} - \alpha_n)D \tag{6}$$

and with (3)

$$y_{n+1} - 2y_n + y_{n-1} = -\frac{s_n}{f} D. \tag{7}$$

With the exception of y_2, which is obtained from (4) and (3), all the y's can be successively determined with (7). The equations which relate the beam excursions to the displacements of the beam centers relative to the lens centers are thus as follows:

$$\begin{aligned}
y_2 - y_1 &= -s_1 \frac{D}{f} + \alpha_1 D \\
y_3 - 2y_2 + y_1 &= -s_2 \frac{D}{f} \\
&\vdots \\
y_{n+1} - 2y_n + y_{n-1} &= -s_n \frac{D}{f} \\
&\vdots \\
y_N - 2y_{N-1} + y_{N-2} &= -s_{N-1} \frac{D}{f} \cdot
\end{aligned} \tag{8}$$

Case 1

We consider first the case that the laser beam and the entire guide are aligned with the exception of one lens.

The beam proceeds on a straight path until it reaches the misaligned lens (No. i). If the reference line is chosen through the centers of the aligned lenses, *i.e.*,

$$x_n = \begin{matrix} 0 & \text{for} & n \neq i \\ x_i & \text{for} & n = i \end{matrix} \tag{9}$$

then with (1)

$$s_n = \begin{matrix} 0 & \text{for} & n < i \\ -x_i & \text{for} & n = i \\ y_n & \text{for} & n > i \end{matrix} \tag{10}$$

(8) reduces to

$$\begin{aligned}
y_n &= 0 \quad \text{for} \quad n \leq i \\
y_{i+1} &= x_i \frac{D}{f} \\
y_{n+2} - y_{n+1}\left(2 - \frac{D}{f}\right) + y_n &= 0 \quad \text{for} \quad n \geq i.
\end{aligned} \tag{11}$$

The general solution of above difference equation can be written as follows:

$$y_n = A\left(\frac{p}{2} + j\sqrt{1 - \frac{p^2}{4}}\right)^{n-i} + B\left(\frac{p}{2} - j\sqrt{1 - \frac{p^2}{4}}\right)^{n-i}, \tag{12a}$$

with $n \geq i$ and

$$p = 2 - \frac{D}{f} \cdot \tag{12b}$$

The coefficients A and B are determined by

$$y_i = 0 \quad \text{and} \quad y_{i+1} = x_i \frac{D}{f},$$

$$A = -B = -jx_i \frac{D}{2f} \cdot \frac{1}{\sqrt{1 - \frac{p^2}{4}}} \tag{13}$$

If we introduce for $p/2$ the trigonometric function

$$\cos \phi = \frac{p}{2} = 1 - \frac{D}{2f}, \left(0 \leq \phi \leq \frac{\pi}{2}\right) \tag{14}$$

we obtain

$$A = -B = -jx_i \frac{D}{2f} \cdot \frac{1}{\sin \phi} \tag{15}$$

$$\begin{aligned}
y_n &= -jx_i \frac{D}{2f} \cdot \frac{1}{\sin \phi} \left[e^{j(n-i)\phi} - e^{-j(n-i)\phi}\right] \\
&= x_i \frac{D}{f} \cdot \frac{\sin (n-i)\phi}{\sin \phi} \cdot
\end{aligned} \tag{16}$$

The maximum excursion of the beam which may occur because of the displacement of a lens is, therefore,

$$|y|_{\max} \leq x_i \frac{D}{f} \cdot \frac{1}{\sin \phi} = x_i \cdot \frac{2D}{\sqrt{4fD - D^2}} \cdot \tag{17}$$

In the case $D = 2f$, ($\phi = \pi/2$), the maximum beam deflection is $\pm 2x_1$ and one obtains from (16)

$$\begin{aligned}
y_n &= 0 \quad \text{for all even values of } (n - i) \\
y_n &= \begin{cases} +2x_1 & \text{for } n - i = 1, 5, 9, \cdots \\ -2x_1 & \text{for } n - i = 3, 7, 11, \cdots. \end{cases}
\end{aligned} \tag{18}$$

This case is illustrated in Fig. 6. It is the case where the field diameter of the beam is a minimum for a given iteration path length D.

Case 2

Now we assume that the entire guide is aligned and only the laser is out of alignment. Choosing again a reference line which connects all the lens centers, we have $x_n = 0$

$$s_n = y_n \quad \text{for every } n. \tag{19}$$

y_1 and α_1 are given by the laser displacement. (8) reduces to

$$y_2 = y_1\left(1 - \frac{D}{f}\right) + \alpha_1 D \tag{20}$$

$$y_{n+2} - y_{n+1}\left(2 - \frac{D}{f}\right) + y_n = 0. \tag{21}$$

The general solution of (21) has already been given in (12). If the arbitrary number (i) is chosen to be ($i=1$), then the constants A and B are determined by the following conditions at lenses 1 and 2.

$$A + B = y_1 \tag{22}$$

$$\alpha_2 = \alpha_1 - \frac{y_1}{f} \quad [\text{see (3)}], \tag{23}$$

or with (4) and (12a)

$$\begin{aligned} y_2 &= y_1\left(1 - \frac{D}{f}\right) + \alpha_1 D \\ &= A\left(\frac{p}{2} + j\sqrt{1 - \frac{p^2}{4}}\right) \\ &\quad + B\left(\frac{p}{2} - j\sqrt{1 - \frac{p^2}{4}}\right). \end{aligned} \tag{24}$$

Eqs. (22) and (24), with (12b), yield

$$A - B = -j\frac{1}{\sqrt{1 - \frac{p^2}{4}}}\left(\alpha_1 D - \frac{D}{2f}\, y_1\right). \tag{25}$$

Replacing p by $2\cos\phi$, y_n [see (12)] can be written

$$y_n = (A + B)\cos(n-1)\phi + j(A - B)\sin(n-1)\phi. \tag{26}$$

Inserting the values of (22) and (25) for $(A+B)$ and $(A-B)$, y_n becomes

$$\begin{aligned} y_n &= y_1\cos(n-1)\phi \\ &\quad + \frac{\sin(n-1)\phi}{\sqrt{1 - p^2/4}}\left(\alpha_1 D - \frac{D}{2f}\, y_1\right). \end{aligned} \tag{27}$$

The beam excursion caused by the misalignment of the laser varies again between limits which depend on α_1 and y_1.

Case 3

If the axis of the laser beam is not pointing toward the center of the last lens of the guide, and the laser and this lens are not adjustable, the beam may be bent gradually without increasing the transmission loss. If the required bending is uniformly distributed over all the lenses, then $s_n = s$ is the same for all the lenses. To determine s, we assume the reference line through the centers of the first and the last lens so that $y_1 = y_N$. Then for reasons of symmetry,

$$y_n = y_{N-(n-1)}. \tag{28}$$

We can write (8) in the general form

$$y_n - 2y_{n-1} + y_{n-2} = -s\frac{D}{f} \qquad \text{for } 2 \le n \le N, \tag{29}$$

where

$$y_0 = y_1 - \alpha_1 D. \tag{30}$$

The general solution of (29) is

$$y_n = -\frac{n(n+1)}{2}\, s\, \frac{D}{f} + An + B. \tag{31}$$

The constant B is arbitrarily chosen $B=0$. Any other value for B would simply shift the reference line parallel to itself by the amount B. A and s are determined by the conditions

$$y_N = y_1;\; y_2 - y_1 = \alpha_1 D - s(D/f), \text{ (see 8)},$$

which yield:

$$A = \alpha_1 D\left(\frac{2}{N} + 1\right); \qquad s = \frac{2\alpha_1 f}{N}. \tag{32}$$

Thus, the coordinates y_n of the beam are

$$y_n = \alpha_1 Dn\left(1 - \frac{n-1}{N}\right), \tag{33}$$

and the coordinates x_n of the lenses

$$x_n = \alpha_1 D\left(n - \frac{n(n-1)}{N} - \frac{2f}{DN}\right). \tag{34}$$

As $y_1 = \alpha_1 D$ [see (33)], the axis of the laser beam intercepts the reference line at a distance D from the first lens. Similarly, the guided beam would intercept the reference line beyond the last lens also at the distance D from this lens.

In practice, it is impossible to establish a straight reference line, by standard surveillance methods, because of the variable atmospheric propagation conditions. However, a simple alignment procedure can be worked out if a probing mechanism is available, to determine the displacement of the beam axis from the optical axis of each lens. This is shown in the following.

Case 4

Assume the laser and all the lenses have been roughly aligned and the corresponding s_n established. If the $(N-1)$ equations of (8) beginning with the last one and proceeding upward are multiplied by the factors $1, 2, 3 \cdots (N-1)$, respectively, and added together, one obtains

$$y_N - y_1 = -\frac{D}{f}\sum_{n=1}^{N-1} s_n(N-n) + (N-1)\alpha_1 D. \quad (35)$$

We assume that the direction of the laser beam and the position of the last lens of the guide are not altered in the alignment procedure. In other words, the desired path of the beam has at each lens a bend of the same magnitude. With respect to the reference line introduced for the previous case, the last lens has the coordinate

$$x_N = \alpha_1 D\left(1 - \frac{2f}{ND}\right).$$

If s_N of the preliminary alignment has been measured, the corresponding y_N is given by the sum of x_N and s_N.

$$y_N = \alpha_1 D\left(1 - \frac{2f}{ND}\right) + s_N. \quad (36)$$

y_1 is not affected by the alignment of the lenses since it is given by the direction of the laser beam and the reference line. According to (33)

$$y_1 = \alpha_1 D. \quad (37)$$

With (36) and (37) inserted into (35), the value of

$$\alpha_1 = \frac{N \cdot \sum_1^{N-1} s_n - \sum_1^{N-1} n \cdot s_n + s_N \cdot f/D}{(N - 1 + 2f/ND)f} \quad (38)$$

can be calculated since all the quantities on the right-hand side of (38) are known.

Eq. (32) then yields then s value to which each lens beginning with lens 1 must be set, to obtain uniform curvature of the beam path. Of course, if the curvature is too large, the direction of the laser beam has to be changed to a smaller α_1.

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